Qualitative–Quantitative Reasoning

Understanding and managing the behaviour of numeric phenomena.

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This paper is about the uniquity and importance of qualitative reasoning about quantitative phenomena. From A–B testing and recommender systems to Covid modelling and climate change, numbers and data drive so much of both the work we do within HCI itself and the applications we work on. Sometimes this requires precise calculations, but often decisions or explanations are more qualitative taking into account general tendencies of numeric systems. This is important for our work within HCI when dealing with quantitative data including dynamic systems, and also as we create tools for those teaching, presenting or working with numeric phenomena.

CCS CONCEPTS • Human-centered computing~Human computer interaction (HCI)~HCI theory, concepts and models

Additional Keywords and Phrases: public understanding of numbers, quantitative methods, big data, complexity

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1 INTRODUCTION

Some aspects of the world can be modelled very precisely and even given closed mathematical formulae: the product of volume and pressure in Boyle's Law for the expansion of a gas, or the logarithmic relation between index of difficulty and time in Fitts' Law for hitting simple targets in a laboratory. Often applying these laws in practice requires a level of understanding not in the mathematics law as such: Boyles Law only applies for slow isothermal reactions where the temperature can equalise and does not apply to very small volumes of gas; Fitts' Law is stochastic, so only works for averages, has constants that depend on individual and muscle groups, is about practiced behaviour, so doesn't apply well for short experiments.

In practice many experts apply these more qualitative understandings of the constraints with ease, often based on professional experience: for example, just how long someone needs to practice before we are happy with a Fitts' Law experiment. Furthermore, the more qualitative version of Boyles' law, that gas is compressed under pressure continues to hold even with rapid adiabatic compression, and 'small targets take more time' is more robust in more complex situations than the precise logarithmic formulation of Fitts' Law. This even applies to more theoretical results: the central limit theorem and variations of it assert that with very precise conditions many phenomena tend to average towards a Normal Distribution, but it would be exceedingly rare for a practicing statistician to verify those precise conditions.

These mixed qualitative–quantitative (QQ) methods are often applied tacitly, from back of the envelope calculations, to semi-formal arguments, but rarely acknowledged explicitly. Yet they form the backbone of most scientific understanding. Furthermore, it is these models that allow generalisation from empirical data, allowing us to reason about which aspects are likely to hold beyond our particular experimental conditions and choice of subjects.

In the wider public discourse, we are in a world where large numbers and complex interactions are common including socio-economic issues such as migration and Brexit, environmental issues such as climate change, and uppermost in many people's minds at the present moment Covid and public health. The critical policy decisions that politicians make and the behaviour and judgements of the general public are driven by attempts to make sense of these complex numeric processes, and yet we often lack to educational and computational tools to address these issues.

Qualitative–quantitative reasoning is important in HCI for two reasons:

- ***QQ in HCI*** – There are various phenomena in HCI, for example motor tasks, where QQ reasoning can be helpful to understand and design for more effective human interaction.
- ***HCI of QQ*** – Understanding QQ reasoning is important in society on general and so there are HCI challenges in designing effective systems for those needing to reason about or communicate about QQ phenomena.

We'll look at each in a little more detail in this paper. The overall aim is to highlight the importance of these computational and mathematical models, which are often tacit, and to encourage more explicit formulation and discussion in scientific work in HCI.

2 ABOUT QUALITATIVE–QUANTITATIVE REASONING

2.1 Why worry anyway?

Sometimes we need a precise answer. On 23rd September 1999 contact was lost with the Mars Climate Orbit as it manoeuvred into orbit around the Red Planet; subsequent investigation established this was due to a mix-up between Imperial and metric units [Ho99, Si18].

However, often it is sufficient to have a less precise model: for example (i) sending a rocket faster puts it in a higher orbit; (ii) water boils at lower temperature when pressure drops; (iii) shortage of labour increases wage costs. In example (i) and (ii), we could have an exact model $G=V^2R$ for (i) or using the phase transition diagram for water in Figure 1. However, for many purposes, especially in explaining phenomena, or making early design decisions, the Q–Q knowledge is sufficient: if we want to vapourise water we either increase temperature or reduce pressure.

In the case if (iii), although this is fundamentally a quantitative problem, we are not going to be able to produce a precise predication of exactly how strong the effect might be, at best some approximate idea based on past experience, or back-of-the envelope calculations. The lack of a precise numerical formula does not mean that it is impossible to have critical discussions or make decisions based on numeric arguments, especially where one has order-of-magnitude or more precise estimates.

2.2 A personal introduction to QQ

I was personally fortunate in having explicit exposure to QQ reasoning early in my career.

My first job was working in mathematical and computational modelling of electrostatically charged agricultural sprays. Even today, with exascale computing, it would be a challenge to model the precise shape of thousands of blades of grass moving in the wind and the electrostatic effects mediated by sap, membranes and soil properties. With computers 100 million times slower and smaller than today, the models were simplistic in the extreme: consisting of a few thousand points on flattened two dimensional views of the crop.

It was evident that the model was no in any way a precise simulation of the actual situation and that it could not be used to predict exactly what charge or spray rate would lead to a given pattern of coverage. The only way to deal with this data was to use the precise but simplistic computational model to build a qualitative understanding of the

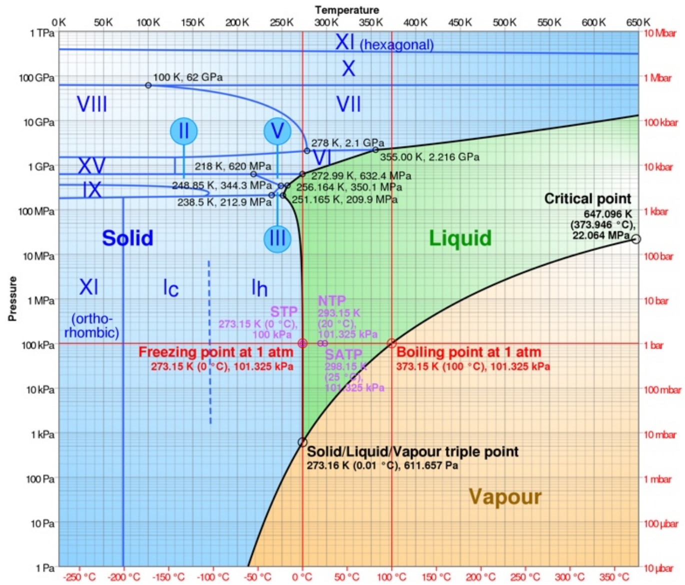

Figure 1: Phase diagram of water. By Cmglee - Own work, CC BY-SA 3.0,, via Wikimedia Commons.
(https://commons.wikimedia.org/wiki/File:Phase_diagram_of_water.svg)

phenomena and then to apply this qualitative model to decision making on real systems [DM84]. Crucially these qualitative understandings sometimes contradicted 'common sense' expectation, for example, increasing charge on the spray could in certain circumstances reduce the amount being prematurely deposited on the top (and apparently most electrostatically active) part of the crop.

Furthermore, while copious computation suggests including everything in a model, limited resources encourages discernment. One quickly became adept at performing order of magnitude assessments of curvature and surface tension, electrostatic attraction and fluid momentum. The aim of these was not to yield precise answers, but to see which elements were worth including in more detailed modelling. If, say, surface tension effects were several times smaller than momentum and electrostatic forces, then only the latter need be included in a mathematical model or computational simulation.

These sound like simply war stories from a different era, but are precisely some of the lessons that are essential currently for Covid modelling.

2.3 Understanding Covid

Early in the Covid crisis I wrote about some of the reasons it was so hard for decisions makers and the general public to understand the pandemic, and the ensuing challenges this raised for HCI [Dx20].

Some of the issues were about the nature of reasoning and cognitive bias in general, for example the tendency towards dichotomous reasoning: notably a focus on ideas of safe vs not safe rather than degrees of risk. However, the numerical issues and in particular QQ reasoning are also critical. Early in the pandemic the shear rate of growth of exponential phenomena left many confused. The lags due to incubation periods and the progression from symptoms, to critical illness and in some cases death, also made it hard to 'read' the figures. This has also been a feature of more recent surges.

Even more difficult has been the ability to see the relationships between small, apparently insignificant actions, are larger scale impact. This can lead to a viewing of the disease as a purely external phenomenon, rather than one that is linked to personal and governmental actions. The discussion of second and third waves is often couched in these terms. In the UK in February there were statements about being passed the peak (of the post-Christmas wave), which then led to pressures for relaxation, as if the wave were a storm that had passed of its own accord rather than decaying because of restrictions.

The scientific models being used for Covid are increasingly sophisticated and detailed, however, they inevitably diverge both because they are fundamentally stochastic and because none of them capture every aspect. The search for a single complete model, like Borges map is doomed to failure [Bo46]. The UK SAGE (Scientific Advisory Group for Emergencies) advice to government takes multiple models, combines these to create a range of estimates and then converts these into a 'traffic' like textual assessment low/medium/high for impact and confidence. Table 1 shows an example of this for Higher Education with a spread of increases in R of "*0.2–0.5*" (corresponding to at least quadrupling societal cases over period of a term) and the textual assessment "*Moderate impact (high confidence)*" (N.B. highest of any intervention except full lockdown).

Table 1: SAGE assessment of impact of University closure on Covid-19 transmission. [SAGE20]

Intervention:	Closure of Higher Education
Impact on COVID transmission:	Moderate impact (high confidence). Outbreaks are very likely in universities, given their size and the degree of close contact typical through shared living arrangements and while socialising and during lectures and practicals. Universities associated with outbreaks of other diseases (e.g. mumps and meningitis) and clear evidence from the US of transmission of COVID in this setting. Closing universities associated with a ~0.3 (0.2-0.5) reduction in the R number. Mitigations short of closure should include strong steer towards online learning for all but essential practical activities.

Crucially note that SAGE are converting these precise (but not fully representative or accurate) quantitative models and recasting them is a more qualitative way that treats them as obtaining evidence, but not precise predictions about the future course f the disease or the impact of interventions. Similar outcomes are seen in climate change modelling, with relatively wide variation, but where the confluence of multiple models using different techniques and assumptions gives rise to a higher degree of confidence than any single model on its own, no matter how complete.

The problems that SAGE and climate scientists face is that presenting this kind of data with all its messiness can open the door to sceptics, both outside and within the scientific community, as I found myself during last summer. SAGE deliberately retain a level of vagueness rather than presenting the full details of the models considered, probably in part to prevent nit-picking attacks.

3 QQ REASONING IN HCI

This workshop is all about complexity theory in HCI, and so not surprisingly, there are many uses or potential uses of qualitative reasoning about quantitaive processes within HCI. Indeed, my recent book on Statistics for HCI is precisely focused on giving this deep understanding rather than numerous specialised tests. [Dx20b].

The most obvious is Fitts Law and related motor level work. Fitts original paper [Fi54] ascribes the logarithmic relationship to information capacity of the human brain, following on from the remarkable success of Shannon and Weaver's information theory in communications [Sh48, SW63]. This information limit arises naturally from a cybernetic modelling of target acquisition as a dynamic closed-loop feedback system (see simplified model Figure 2). Arm and hand movements tend to have an inaccuracy approximately proportional to the distance moved, this naturally gives rise to an (on average) logarithmic number of steps to target. Understanding this model means it is often possible to predict the outcomes of experiments.

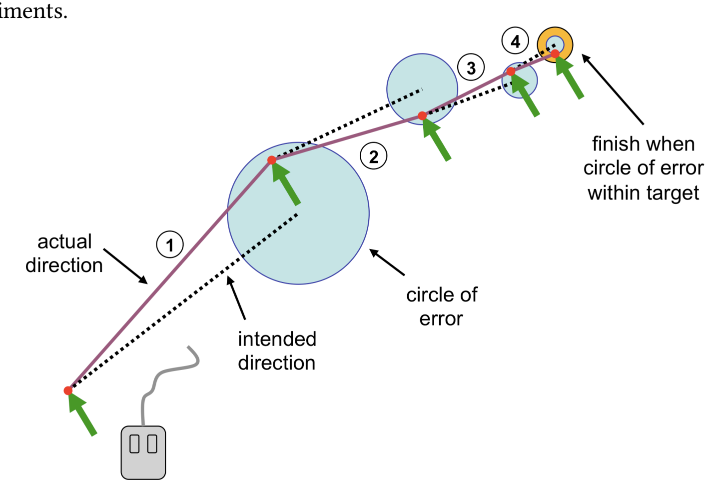

Figure 2: Cybernetic model of Fitts' Law (from [Dx03]). Note (i) not too scale: the first ballistic stage is often around 90% of distance to target; (ii) the number of steps is stochastic: the logarithmic rule is an average; (iii) this is shown as discrete steps, but (time delayed) visual feedback in reality gives amore continuous pattern of movement and correction.

Note that target acquisition includes a number of aspects of dynamic systems all of which are complex in their own right:

- ***Closed-loop feedback*** – Target acquisition depends on hand-eye coordination, with the visual system allowing successive refinement of pointer position. On its own our movements are sufficient to accurate to the size of their effector (arm accurate to hand-sized target, wrist to fingertip), more accurate movements depend on the feedback.
- ***Time delays*** – The feedback loop through the visual system to motor commands and then messages to muscles takes several hundred milliseconds. This means that additional delays slow down acquisition proportionate to this base delay.
- ***Stochastic behaviour*** – The 'error' between planned movement and ballistic movement is probabilistic, so that, rather like a game of golf, sometimes several corrective movements are required, sometimes there is a 'hole in one' – the logarithmic time is an average.

Sadly, the HCI literature in this area is largely based on theory-free empirical studies, but the psychological literature has developed substantially since the time of Paul Fitts.

Other larger-scale examples of QQ reasoning within HCI include importance–cost trade-offs for bug fixing or feature enhancement, and Grudin's seminal work on critical-mass issues in CSCW [Gr88]. The latter was originally developed as post-hoc analysis of existing success and failure stories, but was then used subsequently by Andy Cockburn to drive design decisions [Co93]. Cockburn took the critical mass diagram (Figure 3) and used it to derive potential interventions such as increasing the zero-point value.

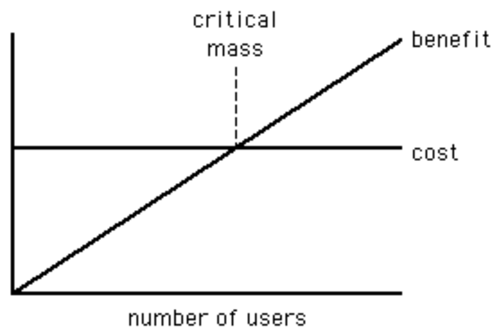

Figure 3: Critical mass in CSCW systems. Value for each user grows with the number of users, but cost is relatively fixed. How do you get the first few users? [Gr88]

During dot-com era colleagues and I were faced with finding ways to encourage growth without massive advertising budgets. This was before ideas such as viral marketing were mainstream, so had to derive our own notions of market ecologies and lattice of value, analysing potential market domains for self-reinforcing feedback loops between different groups of potential users (e.g, teachers, pupils, parents for educational domain) and then seeking design interventions to strengthen critical pats or establish new ones. This was based fundamentally on the qualitative understanding of dynamic systems.

4 HCI FOR QQ REASONING

We have seen many examples complex phenomena that are important in understanding Covid: time delays, feedback loops and exponential growth, accumulation of large numbers of small effects. In addition, these are often complicated by stochastic elements and long-tail effects.

A good example of the difficulty of this for Covid was early growth figures outside China. The week-on-week cases were approximately doubling every week in the earliest days, compared with even more rapid growth during the earliest recorded periods in Wuhan (nearer 5 times increase per week). This led to the impression of a slower growth rate, perhaps due to different infection risks. However, the non-China cases were being driven by international travel not community transmission, and simply reflected the slower growth rate in China once additional heath measures had been put in place.

Exactly the same misapprehension has been repeated recently when looking at school transmission, as the rates of growth within schools are only marginally higher than those outside. However, these matching of growth between coupled systems is precisely what one expects even if one system is driving the other. For schools this is not evidence that they are major drivers, but also not evidence, as is often assumed, that they are not.

The challenge we have in HCI is how to help create means to communicate these kinds of effects to the public and also how to help those tasked with driving policy to model them in ways that are helpful. There are specific educational visualisations, such as some of the animations of exponential growth heavily used during early days of Covid, or Swiss cheese analogies used in safety analysis. There is still an educational challenge in making these more interactive and compelling. The next step is to turn these into more professional tools that go beyond understanding situations are complex to ways to make specific predictions and plans.

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